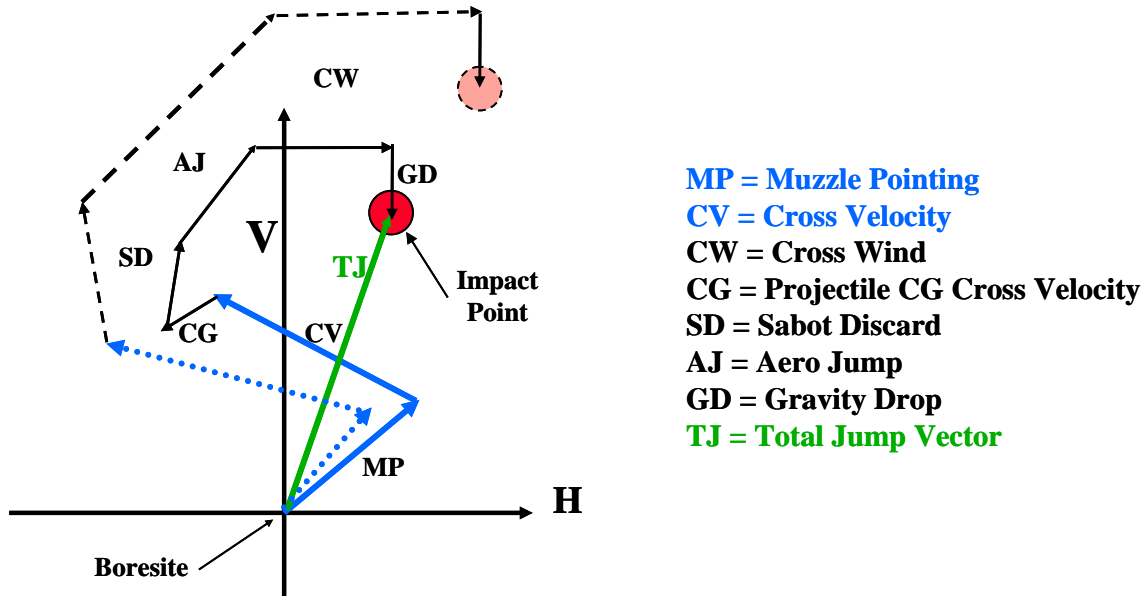


Using PRODAS 6DOF Trajectory Simulation to Estimate Dispersion

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We frequently receive requests to estimate dispersion of a projectile at specific ranges based on a defined error budget. A typical error budget for two shots is shown in Figure 1.

Figure 1: Error Budget Example

For direct fire applications versus a vertically oriented target, it is quite simple to use the fixed plane 6DOF trajectory simulation in PRODAS to generate a series of sensitivity factors as a function of range, multiply by each of the appropriate sized error factors, and add the components up in a root-sum-square fashion to estimate the total dispersion of the system at any range desired.

In Figure 1, the barrel related components are shown in blue, while the dispersion components related to the projectile are in black. The barrel related components, muzzle pointing and cross velocity, are typically fairly small, provided the in-bore transit time of the projectile is reasonably repeatable (standard deviation of approximately 0.3 msec or less). For this reason, and because the barrel related dispersion components are typically much smaller than the other components once the bullet has traveled a few hundred yards (meters) down range, we typically ignore these components of the dispersion error budget.

The effect of the remainder of the error budget elements on dispersion can be simulated relatively simply, by running a series of 6DOF trajectory “perturbations” and subtracting those results from a “baseline” 6DOF trajectory. The trajectory perturbations typically run are:

- + 50 rad/sec Pitch Rate
- + 50 rad/sec Yaw Rate
- + 10m/sec muzzle velocity
- + 2% drag
- + 10 m/sec cross wind
- + 10 m/sec head wind

When performing the perturbation trajectory simulations, one must be mindful to “restore” the perturbed initial conditions to their baseline values when running subsequent simulations. The tabulated output information is copied to a spreadsheet for subsequent operations and summations.

Once the baseline and perturbation trajectory simulations have been completed, the generation of what we call “sensitivity factors” in the horizontal and vertical plane at each output range by subtracting the Y and Z values for the baseline trajectory from the perturbed trajectory and dividing by the input perturbation value (e.g. divide by 50 for a 50 rad/sec input yaw rate) to get a sensitivity factor in inches or mm per unit perturbation factor. The generated sensitivity factors are then multiplied by the estimated error magnitude for each dispersion component (1 sigma) to generate an error at range for each dispersion source. An example of the dispersion estimation procedure is shown below.

It is typical to put the one standard deviation error budget values at the top of a spread sheet work sheet.

	1 σ value
Angular rate	10 rad/sec
Cross Wind	2.37 m/s
Head Wind	2.37 m/s
Drag Variability	1.5 Percent
MV Variability	20 FPS
Range Estimation	25 m
Aim Error	0.05 mils

Table 1: Typical Error Budget Component Values

Then start with a “baseline” 6DOF trajectory simulation which has output increments desired and also has an initial gun elevation which allows the projectile to cross the maximum range of interest a few meters above the ground (e.g. Z at last range between 5 & 10 meters). Next, run a series of 6DOF simulations with the “bulletized” perturbations shown above, and subtract these results from the “baseline” simulation to generate “sensitivity factors in the Y & Z axes as a function of range. Table 2 shows the Vertical Jump sensitivity factor and Horizontal Cross Wind Sensitivity factors in the two right hand columns.

Time sec	X m	Y m	Z m	Slant m	Vertical Jump Sensitivity from Cross Wind	Horizontal Drift Sensitivity to Cross Wind
10 MPH X Wind						
0	0	0	0	0	0.0000	0.0000
0.132	99.86	0	5.32	100	0.0000	0.0022
0.264	199.73	0	10.48	200	0.0022	0.0067
0.3961	299.6	-0.01	15.46	300	0.0022	0.0067
0.5282	399.49	-0.02	20.27	400	0.0022	0.0089
0.6602	499.38	-0.04	24.92	500	0.0022	0.0133
0.7923	599.28	-0.06	29.39	600	0.0022	0.0156
0.9244	699.19	-0.08	33.69	700	0.0022	0.0178
1.0565	799.11	-0.11	37.82	800	0.0022	0.0200
1.1886	899.03	-0.14	41.78	900	0.0022	0.0222
1.3216	998.96	-0.17	45.57	1000	0.0044	0.0267

Table 2: Cross Wind Sensitivity Factors

Next return the perturbed input to its original value and proceed to generate the next trajectory and compute horizontal and vertical sensitivity factors. This procedure is repeated until the user has run trajectories for all the components in the dispersion error budget of interest.

Once this is complete, collect the results of the product of the error budget factors multiplied by the sensitivity factors all into one area, and root-sum-square the error budget factor results as shown in Table 3.

Slant Range, m	Ang. Rate, H @ 10 rad/sec	Ang. Rate, V @ 10 rad/sec	X wind, H @ 2.37 m/s	X Wind, V @ 2.37 m/s	Head Wind, H @ 2.37 m/s	Head Wind, V @ 2.37 m/s	Drag, V @ 1.5 Percent	Muz. Vel., V @ 20 FPS	Range Error, V @ 25 m	Aim Error, H @ 0.05 mils	Aim Error, V @ 0.05 mils	Miss S.D., m
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
100	0.020	0.020	0.000	0.005	0.000	-0.005	0.000	0.022	0.005	0.005	0.037	
200	0.040	0.040	0.005	0.016	0.000	-0.016	0.000	0.042	0.010	0.010	0.076	
300	0.060	0.060	0.005	0.016	0.000	-0.016	0.000	0.063	0.015	0.015	0.111	
400	0.080	0.080	0.005	0.021	0.000	-0.021	0.000	0.086	0.020	0.020	0.149	
500	0.100	0.100	0.005	0.032	0.000	-0.032	0.000	0.108	0.025	0.025	0.189	
600	0.120	0.120	0.005	0.037	0.000	-0.037	-0.008	0.126	0.030	0.030	0.226	
700	0.140	0.140	0.005	0.042	-0.005	-0.042	-0.007	0.151	0.035	0.035	0.268	
800	0.160	0.160	0.005	0.047	-0.005	-0.053	-0.007	0.172	0.040	0.040	0.309	
900	0.180	0.180	0.005	0.053	-0.005	-0.058	-0.015	0.189	0.045	0.045	0.347	
1000	0.200	0.200	0.011	0.063	-0.005	-0.063	-0.015	0.215	0.050	0.050	0.395	

Table 3: Error Budget Components and RSS Miss

From here, you can plot up total miss vs. range or look at which factors comprise the largest portion of the total error budget (range error and angular rate induced dispersion in the example above).

For further information on using PRODAS to estimate dispersion, please contact Jeff Siewert at jsiewert@prodas.com or 802-865-3460 ext 19.